BARNHART

Determination of the Constants & Errors of a Three Inch Transit Zenith Telescope

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# DETERMINATION OF THE CONSTANTS AND ERRORS OF A 3-INCH TRANSIT AND ZENITH TELESCOPE

BY

CHARLES ANTHONY BARNHART

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THESIS FOR THE DEGREE OF BACHELOR OF ARTS

IN MATHEMATICS

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COLLEGE OF SCIENCE

UNIVERSITY OF ILLINOIS

PRESENTED JUNE, 1905

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May 25 1903

THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

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OF Bachelor of Arts.

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HEAD OF DEPARTMENT OF Mathematics.

75381

# Preface

It is the purpose of this short treatise to show the best methods of observation and computation in the use of a combined transit and zenith telescope for the accurate determination of time and geographical latitude. In order to obtain the desired accuracy for these quantities, an exact knowledge of all constants and errors of the instrument are necessary. This knowledge can only be obtained through an accurate determination of these constants and errors.

The writer has endeavored to make this treatise an essentially practical one and suited to the requirements of beginning work with a transit instrument. To that end, broken transits, curvature of a star's apparent path, etc, have not been treated as they constitute a part of more advanced study than is here contemplated

An endeavor has been made to glean the best methods from recognized authorities, as well as to arrive at new methods of treatment. So far as the

writer knows, the details of the method by which he determines the collimation constant appear in print for the first time. The method seems to be more simple and desirable than those now in use. In several instances, slight improvements have been made to various methods already in general practice. An attempt has been made to introduce methods, due to others, which have not found their way into general practice as the method of computing the transit factors, entirely by means of natural functions by using Crelle.

The methods of reduction are intended to be exact to the extent that none of the value and precision of the observations will be sacrificed in the computations. Further refinement would lead to unreliable results and mislead the inexperienced beginner.

Comstock and Campbell have been the chief authorities consulted by the writer, both of whom have covered the field of practical astronomy in an admirable manner.

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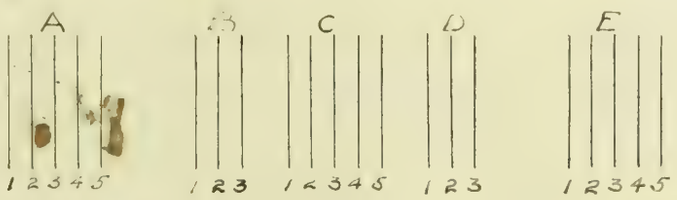
1. Description of the Transit Instrument — The essential parts of a transit are a telescope and a horizontal axis, to which the telescope is attached perpendicularly. The ends of this axis, the pivots, rest in angular notches called wyes. The straight line joining the centers of the pivots is the rotation axis and the straight line, passing through the optical center of the objective and intersecting the rotation axis perpendicularly is the collimation axis. If the telescope is revolved about the rotation axis, the collimation axis describes a plane, known as the collimation plane. This plane should be mounted in the plane of the meridian.

In the common focus of the objective and eye-piece is placed a system of wires called the reticle. These so-called wires, are lines ruled upon glass, which have been filled with plumbago. The wires are arranged in the following manner;




The wires, always odd in number, are placed parallel to

the collimation plane and perpendicular to the rotation axis. The middle wire, $C_3$, is fixed in the collimation plane. The times of transit of a star's image are observed over these wires. Two wires perpendicular to the above wires mark the center of the field of view. A micrometer wire is arranged so as to move parallel to the first set and as nearly as possible in the same plane.

The instrument is arranged so that its rotation axis can be reversed in azimuth by means of a lever. The pivots are raised carefully from the weges by means of the lever. Keeping the lever in this position, the instrument is turned gently through 180°, and the pivots are lowered carefully until they rest firmly in the weges. In this manner, the instrument may be reversed without appreciable jarring. The horizontal axis of the instrument is hollow and two electric light (one for clamp west, and the other for clamp east), the positions of which are adjustable, are placed as nearly as possible in the rotation axis produced. The rays from a light pass through the pivot and fall upon a small mirror on the side of the telescope, which reflects them toward the eye piece, making the wires appear as dark lines in a bright field. The lights should be arranged so that they will not heat the instrument appreciably. The level, in this case a hanging level, is suspended from the horizontal axis. The mi-

rometer box can be rotated through 90° to make the movable wire either perpendicular or parallel to the rotation axis. A diagonal eye-piece enables the instrument to be used on zenith stars.

A coarse circle, with 30′ divisions, is used for setting the telescope at the approximate zenith distance of the star to be observed. With the telescope in this position, an accurate setting is obtained by using a smaller circle with 10′ divisions and with two verniers reading to the nearest 10″. These vernier arms are attached to a delicate [illegible] one of the verniers has been set at the proper reading for the star, the [illegible] is moved until the [illegible] The star will then pass the field of the eye-piece and can be made to pass between the two [illegible] wires by turning the telescope in zenith distance by means of the slow-motion [illegible]

2. Theory of Transit — If a transit instrument is in perfect adjustment, the rotation axis lies in the prime vertical; the horizon and the middle wire lie in the collimation plane. But since such an adjustment is impossible in practice, there are always three errors of adjustment that must be determined before any reliable work can be done with the instrument. These are (1), the azimuth constant a, the angle which the rotation axis makes with the prime vertical and which is positive when the west end

of the axis is too far south; (2) the level constant, $b$, the angle which the rotation axis makes with the horizon and which is positive when the west end of the axis is too high; and (3) the collimation constant, $c$, the amount by which the angle between the line of sight and the west half of the rotation axis exceeds 90°. The line of sight means the imaginary line passing through the optical center of the objective and the middle transit wire.

A fixed relation always exists between the above instrumental errors and the time, $T$, at which a star crosses a given wire. This relation is readily determined.

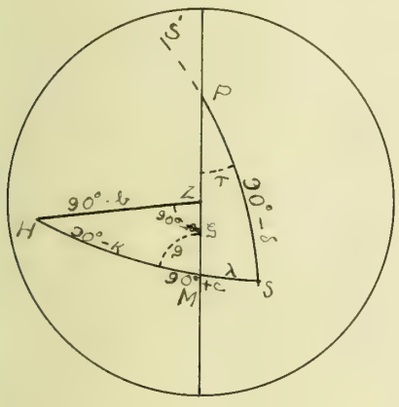



Fig. 1.

Fig. 1 represents a projection of the celestial sphere upon the plane of the horizon. Z is the projection of the zenith, P of the pole, H of the point on which the west half of the rotation axis pierces the sphere, and S is the projection of a star observed just as it is crossing a wire, the angular distance from H of which is 90°+c. (In the figure a, b and c represent the errors discussed above). $\tau$ represents the hour angle of the star reckoned toward the east, $\delta$ is the star's declination, 90°−$\kappa$ is the arc HM, and $\lambda$ is the distance of the star from the meridian measured along HS. It must be noted that the above symbols represent the actual magnitudes of the arcs and angles on the sphere.

From the spherical triangle MSF, we obtain,

$$\cos\delta \sin\tau = \sin\lambda \sin\vartheta$$

and in the triangle MHL

$$\sin k = \sin b \cos\xi + \cos b \sin\xi \sin a$$

The sines may be replaced by arcs whenever a and b are sufficiently small that their cubes and higher powers are negligible (say, less than 5°). Assuming that the instrument with which we have to deal is adjusted, we may put $\sin\vartheta = 1$ and $\xi = \Phi - \delta$, where $\Phi$ is the latitude of the observer. The above equations then would take the following forms

$$\tau = \lambda \sec\delta$$

$$k = b\cos(\Phi-\delta) + a\sin(\Phi-\delta)$$

From the figure

$$90° + c = 90° - k + \lambda$$

or

$$\lambda = c + k$$

$$= c + b\cos(\Phi-\delta) + a\sin(\Phi-\delta)$$

Therefore

$$\tau = c\sec\delta + b\cos(\Phi-\delta)\sec\delta + a\sin(\Phi-\delta)\sec\delta.$$

But $\tau$ is an east hour angle, and bears the following relation to T, the observed time, $\Delta T$ the chronometer correction and $\alpha$ the star's right ascension

$$T + \Delta T = \alpha - \tau$$

which gives by the elimination of $\tau$, Mayer's equation of the transit instrument.

$$\alpha - T = \Delta T + \sin(\Phi-\delta)\sec\delta \cdot a + \cos(\Phi-\delta)\sec\delta \cdot b + \sec\delta \cdot c$$

or

$$\alpha - T = \Delta T + Aa + Bb + Cc.$$

where

$A = \sin(\phi - \delta)\sec\delta$, $B = \cos(\phi - \delta)\sec\delta$ and $C = \sec\delta$.

It should be noted that $(\phi - \delta) = z$, the zenith distance of the particular star, which is being observed.

The coefficients A, B and C are called transit factors, and when, as at an observatory, many observations are to be made at the same latitude, their values are tabulated with the declination as argument. From such tables, the value of these factors can be interpolated for any star.

We must also take into consideration the effect of diurnal aberration, which is to throw the star east of its true position. As a result, it is observed too late and the time of observation must be diminished by the quantity.

$$0''.31 \cos\phi \sec\delta \text{ or } 0^s.021 \cos\phi \cdot c$$

The formula giving the value of $\alpha - T$ was deduced for a star at upper culmination. Now, if A', B' and C' be the transit factors for a star at lower culmination

$$A + A' = 2\sin\phi, \quad B + B' = 2\cos\phi, \quad C + C' = 0.$$

A', B' and C' may be derived from the table of transit factors by making use of these equations. The factors are found to have the following algebraic signs for places in the northern hemisphere:

| Factor | A | B | C |
|---|---|---|---|
| [illegible] with | + | + | ± |
| [illegible] with to pole | − | + | ± |
| Below pole | + | − | ∓ |

All the factors have opposite signs above and below the pole due to the movement of the star in opposite directions. The double sign of C is due to the fact that the reversal of the instrument changes the sign of the collimation constant. It is customary to ignore this change of sign in c by changing the algebraic sign of C, whenever the instrument is reversed, that is

For clamp west $C(+c) = (+C)c$

For clamp east $C(-c) = (-C)c$.

The form of the transit factors in Mayer's equation of the transit instrument is not the best for the computation of these factors. They can be written in such a form that the computations may be done by means of Crelle, which is more rapid than by logarithms, since we shall have certain constant factors for all arguments of $\bar{t}$. As mentioned above, the form of the transit factors may be changed.

$$A = \sin(\Phi-\delta)\sec\delta = \frac{\sin\Phi\cos\delta - \cos\Phi\sin\delta}{\cos\delta} = \frac{\cos\Phi\sin\Phi}{\cos\Phi} - \cos\Phi\tan\delta = \cos\Phi(\tan\Phi - \tan\delta)$$

$$B = \cos(\Phi-\delta)\sec\delta = \frac{\cos\Phi\cos\delta + \sin\Phi\sin\delta}{\cos\delta} = \frac{\sin\Phi\cos\Phi}{\sin\Phi} + \sin\Phi\tan\delta = \sin\Phi(\cot\Phi + \tan\delta)$$

and as before

[1] Herman S. Davis. Popular Astronomy, 19 : 2 p. 303.

$C = \sec \delta$

The computations may be arranged as shown in the table. It is to be noted that none of the intermediate steps in the computation need be discarded for the coefficients for Bessel's and Hansen's formsfor reduction are given also.

## Transit Factors

| δ | cos δ | tan Φ − tan δ | cot Φ + tan δ | A | B | C |
|---|---|---|---|---|---|---|
| 15°00′ | 0.268 | 0.574 | 1.455 | 0.434 | 0.431 | 1.035 |
| 20′ | 0.274 | 0.568 | 1.461 | 1.435 | 1.434 | 1.037 |
| 40′ | 0.281 | 0.561 | 1.468 | 1.429 | 1.446 | 1.039 |
| 6°00′ | 0.287 | 0.555 | 1.474 | 0.425 | 0.949 | 1.041 |
| 20′ | 0.293 | 0.549 | 1.480 | 1.421 | 1.453 | 1.042 |
| 40′ | 0.299 | 0.543 | 1.486 | 1.415 | 1.457 | 1.044 |
| 17°00′ | 0.306 | 0.536 | 1.493 | 0.410 | 0.962 | 1.046 |
| | | | | | | |
| 30°00′ | 0.577 | 0.265 | 1.764 | 1.208 | 1.136 | 1.155 |
| 0′ | 0.581 | 0.261 | 1.768 | 1.204 | 1.139 | 1.157 |
| 20′ | 0.585 | 0.257 | 1.772 | 1.199 | 1.141 | 1.159 |
| 30′ | 0.589 | 0.253 | 1.776 | 1.194 | 1.144 | 1.161 |
| 40′ | 0.593 | 0.249 | 1.780 | 1.189 | 1.146 | 1.163 |
| 50′ | 0.597 | 0.245 | 1.784 | 1.184 | 1.149 | 1.165 |
| 31°00′ | 0.601 | 0.241 | 1.788 | 1.184 | 1.152 | 1.167 |
| | | | | | | |
| 55°00′ | 1.428 | −0.586 | 2.615 | −0.448 | 1.685 | 1.743 |
| 05′ | 1.433 | −0.591 | 2.624 | −1.452 | 1.684 | 1.741 |
| 10′ | 1.437 | −0.595 | 2.632 | −1.455 | 1.682 | 1.739 |
| 15′ | 1.441 | −0.599 | 2.640 | −0.458 | 1.683 | 1.734 |
| 20′ | 1.446 | −0.604 | 2.650 | −1.464 | 1.681 | 1.732 |
| 25′ | 1.450 | −0.608 | 2.658 | −0.465 | 1.679 | 1.730 |
| 30′ | 1.455 | −0.613 | 2.668 | −0.469 | 1.677 | 1.766 |
| 35′ | 1.460 | −1.618 | 2.678 | −0.473 | 1.705 | 1.769 |
| 40′ | 1.464 | −0.622 | 2.686 | −0.476 | 1.707 | 1.771 |
| 45′ | 1.469 | −0.627 | 2.696 | −1.481 | 1.711 | 1.773 |
| 50′ | 1.473 | −0.631 | 2.704 | −1.484 | 1.713 | 1.775 |
| 55′ | 1.478 | −0.636 | 2.714 | 0.488 | 1.716 | 1.778 |
| | | | | | | |
| 70°00′ | 2.741 | −1.905 | 3.719 | −1.451 | 2.535 | 2.924 |
| 02′ | 2.752 | −1.910 | 3.739 | −1.456 | 2.536 | 2.928 |
| 04′ | 2.757 | −1.915 | 3.749 | −1.458 | 2.547 | 2.933 |
| 06′ | 2.762 | −1.920 | 3.759 | −1.460 | 2.543 | 2.936 |
| 08′ | 2.767 | −1.925 | 3.769 | −1.462 | 2.546 | 2.942 |
| 10′ | 2.773 | −1.931 | 3.780 | −1.464 | 2.550 | 2.947 |
| | | | | | | |
| 80°00′00″ | 5.671 | −4.829 | 6.814 | −3.691 | 4.411 | 5.157 |
| 00′30″ | 5.676 | −4.834 | 6.824 | −3.698 | 4.420 | 5.170 |
| 01′00″ | 5.681 | −4.839 | 6.834 | −3.705 | 4.423 | 5.181 |
| 01′30″ | 5.686 | −4.844 | 6.844 | −3.706 | 4.426 | 5.193 |
| 02′00″ | 5.691 | −4.849 | 6.854 | −3.713 | 4.435 | 5.178 |
| 02′30″ | 5.695 | −4.853 | 6.862 | −3.711 | 4.432 | 5.182 |

sin α = 0.64

tan α = 0.842

cos α = 0.765

cot α = 1.18

3. Value of the reduction of the micrometer wire -

The value of one revolution of the micrometer screw must be determined by observations of stars near the celestial equator, which are so important in the observation of transits. The determination of this value may be made by observing a close circumpolar star. The telescope is moved until the star is just visible within the limits of the field of view and will be carried through the middle portion of the field by its diurnal motion. The micrometer wire is set just in advance of the star, for convenience, with a micrometer reading of some multiple of five, and the time of transit of the star across the wire is recorded by means of a chronograph. The micrometer wire is moved forward five revolutions and a second transit observed. The observations are continued until the star has crossed the entire field.

In Fig. 2, let P be the pole, EF the instrumental meridian, M'M'' the diurnal path of the star, AM' one position of the micrometer wire and BM'' any other position of the wire. Let r' be the micrometer reading, t' the hour angle, and T' the sidereal time when the star is at M'; and let r'', t'' and T'' have the corresponding values when the star is at M''. The value of one revolution of the micrometer screw may be represented by r. Through M', describe an arc of a great circle, perpen-

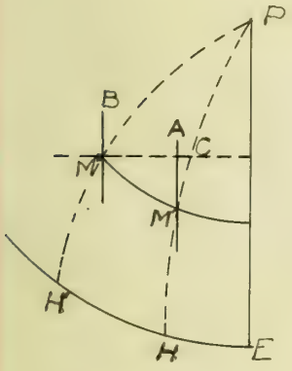



Fig. 2.

dicular to AM'. Then in the triangle CM'P

$$CM'' = (r''-r')R,\quad M''P = 90°-\delta,\quad CPM'' = T''-T' = T''-T';$$

and we can write

$$\sin[(r''-r')R] = \sin(T''-T')\cos\delta$$

or, since $(r''-r')R$ is always a small angle

$$(r''-r')R = \sin(T''-T')\cos\delta$$

and, when the star is not within 10° of the pole

$$(r''-r')R = (T''-T')\cos\delta$$

Then

$$R = \frac{(T''-T')\cos\delta}{r''-r'}$$

For extreme precision, the correction for refraction should be applied to the quantity $(r''-r')R$; but if the observations are made near the meridian, the correction will scarcely be appreciable. If the value of a revolution in seconds of arc is desired, the above expression need only be multiplied by 15.

Example:- 50 Cassiopeiae and 36 H. Cassiopeiae were observed at upper culmination on Dec. 15, 1904 to determine the value of one revolution of the micrometer screw of the transit instrument. 50 Cassiopeiae was observed with the telescope clamp west and with the upper index of the micrometer and 36 H. Cassiopeiae with telescope clamp east and with the lower index.

| r | r'-r' | T | T"-T" | r | r"-r' | T | T"-T' |
|---|---|---|---|---|---|---|---|
| 10 | | $01^h 51^m 36^s.23$ | | 40 | | $02^h 22^m 2[illegible]^s.30$ | |
| 5 | | $51^m 37.74$ | | 45 | | $23^m 26.63$ | |
| 50 | | $53^m 3[illegible]$ | | 50 | | $24^m 2[illegible]$ | |
| 45 | | $54^m 38.49$ | | 5 | | $2[illegible]^m 3[illegible]$ | |
| 40 | | $55^m 39.00$ | | 10 | | $26 33.[illegible]$ | |
| 30 | 30 | $57^m +0.00$ | 363.17 | 15 | | $31.31.6[illegible]$ | |
| 25 | 30 | $[illegible] +0.18$ | 363.14 | 20 | 30 | $[illegible] 31.52$ | $32^s.22$ |
| 20 | 30 | $59^m +1.00$ | 363.20 | 25 | 30 | $29 32.79$ | 312.16 |
| | | | | 30 | 30 | $35 +0.37$ | 311.71 |
| 15 | 30 | $60^m +2.00$ | 363.51 | 35 | 30 | $31 +3.[illegible]$ | 312.[illegible] |
| | | | | 40 | 30 | $32 +5.92$ | 312.66 |
| 10 | 30 | $61^m +2.29$ | 363.29 | 45 | 30 | $33 +7.9[illegible]$ | 313.21 |

Mean 30 Mean 363.38 30 Mean 312.38

$$R = \frac{(T''-T')\cos\delta}{r''-r'}$$

Therefore for 50 Cassiopeiae

$$R = \frac{363.38 \times \cos 71^\circ 57' 50''}{30} = 3^s.7523$$

and for 36 H. Cassiopeiae

$$R = \frac{312.38 \times \cos 72^\circ 24' 15''}{30} = 3^s.7525$$

or the value of one revolution of the micrometer screw is $3^s.752$

4. Determination of Wire Intervals -- If the transit is provided with a micrometer in right ascension, this determination may be made quite easily. Theoretically the micrometer wire should be set on each of the fixed wires in succession, but in practice this can not be done so as to give results of the desired accuracy. More accurate readings can be obtained by setting the micrometer wire on each side of the fixed wire and just in apparent contact with it. The mean of the readings in the two positions is the reading which the coincidences of the two wires should give. The difference of the micrometer readings on the side wires and the middle wire give the wire intervals in terms of one revolution of the micrometer screw, the value of which has already been obtained

Example:— Two sets of micrometer readings were made in each of which the micrometer wire was placed in contact with each side of the fixed wires. The results are given in tabulated form on the next page. A, represents the first wire which a star approaching upper culmination would cross when the telescope is clamped west. The other wires are crossed in the order shown in the description of the transit.

Nov. 16, 1904

| | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ |
|---|---|---|---|---|---|
| | 28.210 | 21.805 | 21.313 | 26.929 | 26.490 |
| | 28.331 | 21.841 | 21.453 | 21.008 | 21.569 |
| | 28.250 | 21.818 | 21.314 | 26.931 | 21.490 |
| | 28.381 | 21.853 | 21.449 | 21.009 | 26.573 |
| Mean | 28.290 | 21.829 | 21.412 | 26.964 | 26.531 |

| | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| | 24.730 | 24.258 | 23.824 |
| | 24.809 | 24.361 | 23.928 |
| | 24.730 | 24.287 | 23.841 |
| | 24.801 | 24.366 | 23.925 |
| Mean | 24.761 | 24.321 | 23.888 |

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---|---|---|---|---|---|
| | 22.113 | 22.533 | 22.091 | 21.649 | 21.211 |
| | 23.054 | 22.609 | 22.168 | 21.702 | 21.284 |
| | 22.117 | 22.535 | 22.091 | 21.653 | 21.217 |
| | 23.051 | 22.608 | 22.166 | 21.721 | 21.287 |
| Mean | 23.013 | 22.571 | 22.129 | 21.687 | 21.248 |

| | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|
| | 20.321 | 19.842 | 19.452 |
| | 20.403 | 19.961 | 19.526 |
| | 20.321 | 19.843 | 19.452 |
| | 20.404 | 19.965 | 19.527 |
| Mean | 20.362 | 19.902 | 19.489 |

| | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ |
|---|---|---|---|---|---|
| | 17.613 | 17.253 | 16.831 | 16.367 | 15.923 |
| | 17.769 | 17.328 | 16.910 | 16.447 | 16.009 |
| | 17.694 | 17.253 | 16.830 | 16.361 | 15.926 |
| | 17.768 | 17.332 | 16.908 | 16.445 | 16.006 |
| Mean | 17.731 | 17.291 | 16.870 | 16.405 | 15.966 |

Then, the difference of reading in terms of one revolution of the micrometer screw between each wire and the middle wire was taken. The first column in each case gives this difference and the second column gives the wire interval, which was obtained by multiplying the difference given in the first column by the value of one revolution of the micrometer screw, 3.752.

| | | | | | |
|---|---|---|---|---|---|
| $A_1$ | 6.161 | 23.12 | $E_5$ | 6.163 | 23.12 |
| $A_2$ | 5.711 | 21.45 | $E_4$ | 5.122 | 21.41 |
| $A_3$ | 5.283 | 19.82 | $E_3$ | 5.260 | 19.74 |
| $A_4$ | 4.840 | 18.16 | $E_2$ | 4.838 | 18.15 |
| $A_5$ | 4.402 | 16.52 | $E_1$ | 4.318 | 16.50 |
| | | | | | |
| $B_1$ | 2.640 | 9.90 | $D_3$ | 2.640 | 9.91 |
| $B_2$ | 2.196 | 8.24 | $D_2$ | 2.200 | 8.25 |
| $B_3$ | 1.769 | 6.60 | $D_1$ | 1.776 | 6.62 |
| | | | | | |
| $C_1$ | 0.881 | 3.32 | $C_3$ | 0.881 | 3.31 |
| $C_2$ | 0.442 | 1.66 | $C_4$ | 0.442 | 1.66 |

5. The Value of a Level Division — The best method by which to determine the value of a level division would be to measure by means of a micrometer, the vertical angle through which a tube must be tipped in order to cause the bubble to pass over a given number of divisions of the scale. The necessary apparatus for this method was not available, so the level was attached to a theodolite with its plane approximately vertical.

Let the instrument be inclined until the vertical axis makes some angle $\gamma_1$ with the true vertical. Now, if the instrument is revolved slowly about its vertical axis, the bubble will move back and forth in the tube and there will be two positions of the instrument, $A_1$ and $A_2$, for which the bubble stands at the middle of the scale. A slight revolution of the instrument to either side of these two positions will cause a correspondingly small movement of the bubble.

Fig. 3, which represents a portion of the celestial sphere adjacent to the zenith Z will be used to determine the relation of the bubble reading to the corresponding circle readings. V is the point in which the inclined vertical axis of the instrument pierces the sphere, and SV is the trace upon the sphere of the plane of the level-

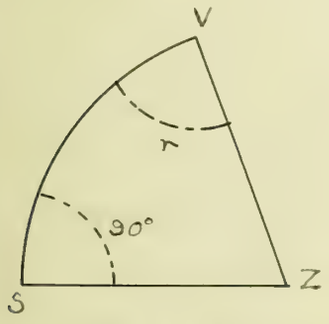



Fig. 3.

tube, which was attached to the instrument with its plane approximately vertical, before the instrument was tipped. As the instrument is rotated, the level-tube moves with it and the arc SV must revolve about V as a pivot. The amount of its rotation will be given by the consecutive readings of the azimuth circle. It is quite evident that the angle r of the figure for any particular reading $A_1$ is given by the equation

$$r = \frac{1}{2}(A_1 + A_2) - A$$

where $\frac{1}{2}(A_1 + A_2)$ is the circle reading for which SV and VZ are coincident.

It is quite easy to see that the bubble always stands at the point of its tube which is nearest the zenith. Therefore, the point in the figure corresponding to the middle of the tube may be found by drawing an arc of a great circle from Z perpendicular to SV. The intersection of this circle with SV will be the required point. Then, in the right spherical triangle ZVS, we have

$$\cos r = \frac{\tan \rho}{\tan y}$$

or

$$\tan \rho = \tan y \cos r$$

in which $\rho$ measures the distance of the middle of the bubble from the fixed point, V.

To find how far the bubble will move when

the instrument is revolved slightly in azimuth, we need only to differentiate the above equation

$$\sec^2 \rho \, d\rho = - \tan y \sin r \, dr$$

$$- b_2 = \tan y \cos^2 \rho \sin r \, dr.$$

Small finite increments of the respective quantities may be substituted in place of the differentials with the following result.

$$2 d (b' - b'') = \tan y \cos^2 \rho \sin r (A' - A'')$$

where $d$ represents the value of half a level division, and $b'$, $b''$ are the readings of the middle of the bubble for the circle readings $A'$ and $A''$ respectively.

This last equation may be employed to determine the value of $d$, but when sufficient care is taken to make the radius passing through the middle point of the scale parallel to the vertical axis of the instrument, the readings $A_1$ and $A_2$ will be so nearly 180° apart that we may put $r = 90°$ and $\cos \rho = 1$ for all positions of the bubble within the limits of the scale and obtain the much simpler relation

$$d = \frac{(A' - A'')}{2 (b' - b'')} \tan y$$

Example;— The axis of the instrument was deflected through an angle of 15' in the following manner. The instrument was carefully leveled and the telescope directed upon the steeple of a building. The telescope was moved through an angle of 15' by

means of the tangent movement to the vertical circle. Then the telescope was brought back to its former position by means of a footscrew. Care should always be taken that the footscrew will cause the telescope to move in the same plane as that in which it was inclined. If this is not done, the telescope can not be directed upon the point chosen when brought back by means of the footscrew. The axis of the instrument will be tipped through an angle of 15' as a result, as the error for such a small angle will be inappreciable.

A'-A" and b'-b" were derived from the readings of the horizontal circle, respectively. In making the observations, the bubble was brought near one end of the tube and the circle set to read some multiple of 5'. Then as the instrument was turned to each successive 5' reading, the readings of the extremities of the bubble were recorded until the bubble had traversed the entire length of the tube, after which the movement of the circle was reversed and readings taken for the same circle settings as before.

It is quite evident that the divisor 2(b'-b") is equivalent to 2 b'-2 b", and since b is the reading for the middle of the bubble, 2 b is equal to the sum of the readings of the ends of the bubble. The differences 2 b'-2 b" were obtained by subtracting the first b from the first

one following the middle of the set, the second b from the second one from the middle, etc

Dec. 21, 1904

| Circle Reading | Forward | | Backward | | | |
|---|---|---|---|---|---|---|
| | E | W | E | W | Σb | 2(b'-b") |
| 0 | 13.5 | 50.7 | 13.1 | 49.5 | 143.4 | |
| 5 | 10.8 | 41.9 | 11.4 | 51.8 | 138.9 | |
| 10 | 88.9 | 46.0 | 89.4 | 45.9 | 135.1 | |
| 15 | 86.8 | 43.9 | 81.3 | 44.0 | 131.0 | |
| 20 | 84.6 | 41.5 | 85.9 | 42.3 | 121.2 | |
| 25 | 83.0 | 39.9 | 84.6 | 41.2 | 124.3 | |
| 30 | 81.3 | 38.2 | 83.0 | 39.3 | 120.9 | |
| 35 | 80.1 | 37.0 | 81.0 | 38.1 | 117.9 | |
| 40 | 18.0 | 34.9 | 19.4 | 36.0 | 114.2 | |
| 45 | 76.0 | 33.0 | 71.0 | 33.5 | 109.1 | |
| 50 | 73.6 | 30.1 | 75.8 | 32.1 | 106.1 | |
| 55 | 72.2 | 28.9 | 73.6 | 30.0 | 102.4 | |
| 60 | 10.1 | 21.4 | 11.4 | 28.0 | 98.7 | 44.7 |
| 65 | 68.6 | 25.3 | 70.1 | 26.9 | 95.5 | 43.4 |
| 70 | 68.0 | 24.8 | 67.7 | 24.2 | 12.3 | 42.8 |
| 75 | 66.1 | 21.9 | 66.1 | 22.7 | 81.9 | 43.1 |
| 80 | 64.3 | 21.2 | 64.2 | 20.5 | 85.3 | 41.9 |
| 85 | 61.8 | 18.1 | 62.1 | 18.8 | 80.1 | 43.6 |
| 90 | 61.2 | 18.0 | 60.1 | 17.3 | 78.6 | 42.3 |
| 95 | 59.3 | 16.0 | 58.6 | 15.2 | 74.5 | 43.4 |
| 100 | 57.7 | 14.5 | 56.9 | 13.5 | 71.3 | 42.9 |
| 105 | 56.1 | 13.4 | 54.8 | 11.6 | 68.3 | 41.4 |
| 110 | 54.0 | 10.6 | 53.0 | 9.0 | 63.3 | 42.8 |
| 115 | 52.8 | 9.2 | 51.3 | 8.1 | 60.5 | 41.9 |

mean 42.85

$\gamma = 15'$ $\quad \log (A' - A'') = 3.5563 \quad \log d = 9.5641$

$A' - A'' = 60' = 3600''$ $\quad \operatorname{colog} 2(b' - b'') = 8.3680 \quad d = 0''.367 = 0^s.024$

$\log \tan \gamma = 7.6398$ $\quad$ Level division $= 2d = 0''.734 = 0^s.049$

6 Determination of the Level Constant — As already defined, the level error, b, is the angle of elevation of the rotation axis above the western horizon. A repetition of the definition was deemed advisable here in order to make the method of this determination more clear. As the zero of the scale was at one end of the tube, and the numbers increased continuously to the other, we shall consider that case. All other cases would be treated in a similar manner.

If the level be placed on a truly horizontal line, the center of the bubble will not be at zero, because of the nonadjusted of the level; for instance, the case where one leg is longer than the other, etc. Suppose the center is n divisions from the zero mark, then the error of the level is $2nd$, where $2d$ represents one level division. Now, let the level be placed in a position forming an angle b with the horizon and let the reading of the west end of the bubble be w and of the east end, e. Then, the elevation of the west end of the line is given by

$$b = \tfrac{1}{2}(w+e)2d \mp 2nd$$

If the level is reversed

$$b = -\tfrac{1}{2}(w'+e')2d \pm 2nd$$

for the west end of the tube would form an angle of opposite sign with the horizon. Combining these two

values of $b$, we have

$$b = \tfrac{1}{2}[(w - e') + (e - w')]\,d$$

A positive value of $b$ will be obtained whenever the west end is higher than the east end. It is to be noticed that the error of the level is eliminated by reversing the instrument. Whenever it is possible the level should be read several times, the same number of readings being made in each position — clamp west and clamp east.

Example:— It was necessary to determine $b$, in some work done on Dec. 5, 1904

| | w | e | w | e |
|---|---|---|---|---|
| Clamp west | 90.3 | 16.2 | 89.1 | 15.0 |
| Clamp east | 9.0 | 83.3 | 9.0 | 85.0 |
| | | 7.1 | | 5.05 |

Mean +6.08

$$b = 6.08 \times 0.^{s}024 = +0.146$$

But the level is applied to the outer surface of the cylindrical pivots and does not give the inclination of the axis, which passes through their centers, unless their radii are equal

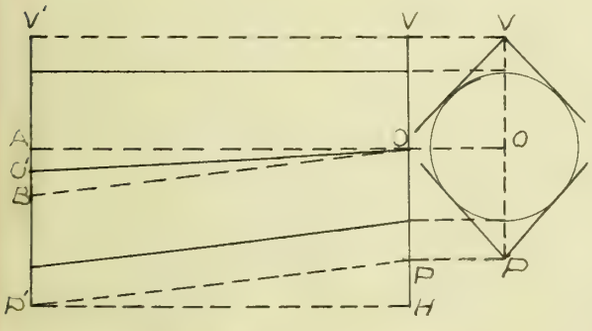



Fig. 4.

To determine the inequality of the pivots, let O and O', Fig. 4, be the centers of the west and east pivots, for clamp west; P and P', the vertices of

of the wyes in which the pivots rest; V and V', the vertices of the wyes of the level; and HP', a horizontal line. Then O'OB = O'OA is the inequality of the pivots which we shall represent by i. If we let

B' = the inclination given by the level for clamp west,

B'' = the inclination given by the level for clamp east,

b' = the true inclination for clamp west,

b'' = the true inclination for clamp east,

β = the constant angle, HP'P.

Then

$$b' = B' + i = \beta - i$$

for clamp west, and

$$b'' = B'' - i = \beta + i$$

for clamp east. Therefore

$$i = \frac{B'' - B'}{4}$$

The value of i was obtained by taking the mean of a number of determinations.

March 22, 1905

| Clamp head | | [illegible] | | B' - B' |
|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | |
| 48.8 | 88.1 | 48.8 | [illegible] | |
| 76.1 | 37.5 | 75.0 | [illegible] | |
| 49.8 | 89.0 | 49.5 | [illegible] | |
| 16.8 | 31.1 | 14.8 | [illegible] | |
| B' = -11.75 d | | B'' = -13.98 d | | -2.23 d |
| 49.7 | [illegible] | 50.1 | 89.6 | |
| 16.9 | [illegible] | 14.9 | [illegible] | |
| 49.7 | [illegible] | 50.7 | [illegible] | |
| 76.1 | 31.7 | 14.1 | [illegible] | |
| B' = -11.58 d | | B'' = -14.85 d | | -3.27 d |
| 50.5 | 88.6 | 51.2 | 89.4 | |
| 15.6 | 31.1 | 13.1 | 38.6 | |
| 50.1 | [illegible] | 51.1 | 89.1 | |
| 14.9 | 37.2 | 13.4 | 38.5 | |
| B' = -12.93 d | | B'' = -15.65 d | | -2.72 d |
| 51.4 | 89.0 | 51.5 | | |
| .6 | 38.8 | 13.3 | | |
| 50.8 | [illegible] | 51.1 | | |
| 14.4 | 37.[illegible] | 13.3 | | |
| B' = -13.83 d | | B'' = -16.03 d | | -2.20 d |

Mean, -2.72 d.

L = 1/4 [illegible], [illegible] = -0.250 = -0.017

## 7. Determination of the Collimation Constant —

The collimation constant is positive when the middle wire is actually ~~east~~ west of the collimation plane. It may be determined by observing the transit of a circumpolar star over a set of wires and then quickly reversing the instrument so as to observe the star's transit over the same set of wires again.

Let $T_1$ represent the time of transit of the star over a certain fixed wire and $T_2$, the time of transit over the same wire after the instrument has been reversed. The difference of the two transits ($T_2 - T_1$) multiplied by the cosine of the declination gives the equatorial time interval between the two transits. This reduction is made because the wire intervals which are involved in the discussion are given in equatorial time intervals. Now, it is evident that

$$T_0 = T_1 + (I + c) \sec \delta$$

and also that

$$T_0 = T_2 + (I + c) \sec \delta$$

By eliminating $T_0$, we obtain

$$c = \frac{T_2 - T_1}{2} \cos \delta - I$$

where $T_0$ represents the time of transit over the middle wire if there is no collimation and I the wire interval of the thread under consideration

Example:- The collimation constant was determined by means of observations made upon 43 H. Cephei at upper culmination and upon Broombridge 2001 at lower culmination on Nov. 23, 1904. As the sign of c is determined with respect to the position of the instrument at the time $t_1$, we find that the collimation constant is negative for both determinations. But as the instrument was clamped east in both cases at the time, $T_1$, the collimation constant would be positive if the instrument were west, the normal position of the instrument

43 H. Cephei $8^{\circ}4'$ (Clamp East.

| Wire | $T_1$ | $T_2$ | $\frac{T_2-T_1}{2}$ | $\log \frac{T_2-T_1}{2}$ | $\log \cos\delta$ | $\log \frac{T_2-T_1}{2}\cos\delta$ | $\frac{T_2-T_1}{2}\cos\delta$ | $\bar{T}$ | $c$ |
|---|---|---|---|---|---|---|---|---|---|
| $E_5$ | $0^h 57^m 02^s.0$ | $1^h 01^m 12^s.1$ | $305^s.55$ | 2.4848 | 9.9911 | 1.3541 | 22.163 | 23.12 | -0.49 |
| $E_4$ | $57^m 55^s$ | $1^m 50^s.9$ | 284.20 | 2.4536 | 9.9911 | 1.3235 | 21.06 | 21.71 | -0.41 |
| $E_3$ | $57^m 47^s.7$ | $1^m 35^s.9$ | 269.10 | 2.4300 | 9.9911 | 1.2834 | 19.20 | 19.74 | -0.54 |
| $E_2$ | $57^m 08^s.9$ | $1^m 06^s.9$ | 239.00 | 2.3784 | 9.9911 | 1.2483 | 17.71 | 18.25 | -0.54 |
| $E_1$ | $57^m 23^s.3$ | $1^h 01^m 02.1$ | 215.15 | 2.3321 | 9.9911 | 1.2026 | 15.94 | 16.50 | -0.56 |

-0.49

Broombridge 2001 $12^{\circ}$ 53' (Clamp West at lower culmination)

| Wire | $T_1$ | $T_2$ | $\frac{T_2-T_1}{2}$ | $\log \frac{T_2-T_1}{2}$ | $\log \cos\delta$ | $\log \frac{T_2-T_1}{2}\cos\delta$ | $\frac{T_2-T_1}{2}\cos\delta$ | $\bar{T}$ | $c$ |
|---|---|---|---|---|---|---|---|---|---|
| $E_5$ | $1^h 22^m 32^s.2$ | $1^h 24^m 51^s.1$ | $76^s.85$ | 1.8856 | 9.9688 | 1.3544 | 22.62 | 23.12 | -0.50 |
| $E_4$ | $22^m 32^s.1$ | $24^m 57^s.6$ | 11.25 | 1.8528 | 9.9688 | 1.3216 | 20.93 | 21.41 | -0.48 |
| $E_3$ | $22^m 38^s.1$ | $24^m 48^s.3$ | 65.10 | 1.8136 | 9.9688 | 1.2824 | 19.16 | 19.64 | -0.48 |
| $E_2$ | $22^m 43^s.2$ | $24^m 43^s.2$ | 60.00 | 1.7782 | 9.9688 | 1.2470 | 17.66 | 18.15 | -0.49 |
| $E_1$ | $22^m 49^s.0$ | $24^m 38.1$ | 54.55 | 1.7364 | 9.9688 | 1.2052 | 16.05 | 16.55 | -0.50 |

-0.50

$c = +0^s.50$ for clamp west.

The collimation constant may also be determined by sighting on a distant terrestrial object. The telescope must be placed in a horizontal position and some well-defined distant point, the image of which is near the middle wire, selected to be employed in the determination. The distances of the image from the middle wire in the two positions of the instrument are measured with the micrometer. This distance will be called D' for clamp west and D" for clamp east. Since the eye-piece reinverts horizontally D' and D", will be positive and negative according as the middle wire is east or west (in the eye piece) of the image. The collimation constant is then given by the following equation

$$c = \frac{1}{2}(D' - D'')$$

for clamp west, the normal position of the instrument.

April 26, 1905

| Clamp | $C_3$ | micrometer on image | micrometer on $C_3$ | Clamp | $C_3$ | micrometer on image |
|---|---|---|---|---|---|---|
| W. | W. of image | 23.320 | 23.180 | E | W. of image | 22.537 |
| | | 23.517 | 23.085 | | | 22.728 |
| | | 23.218 | 23.179 | | | 22.537 |
| | | 23.512 | 23.081 | | | 22.729 |
| | | 23.303 | 23.183 | | | 22.537 |
| | | 23.514 | 23.079 | | | 22.729 |
| | | Mean = 23.411 | 23.131 | | | Mean = 22.632. |

D' = -(23.411 - 23.131) R = - 0.28 R.

D" = -(23.131 - 22.632, R = - 0.499 R.

c = ½[-0.28 R - (-0.499, R] = + 0.11R = + 0.41

8 Determination of the Azimuth Constant — The only method by which this constant can be determined is by observations on the stars. Two stars must be observed; let these stars be represented by $(\alpha_1, \delta_1)$ and $(\alpha_2, \delta_2)$. When the other constants have been determined, we can correct the times of observation of the two stars for all errors except the azimuth. Let $T_1$ and $T_2$ represent these corrected times. Then for the first star observed, we have

$$\Delta T = \alpha_1 - T_1 - a A_1$$

and for the second

$$\Delta T = \alpha_2 - T_2 - a A_2$$

where $A_1$ and $A_2$ are the values of the transit factor A corresponding to $\delta_1$ and $\delta_2$ respectively. Combining the above equations, we obtain

$$a = \frac{(\alpha_1 - T_1) - (\alpha_2 - T_2)}{A_1 - A_2}$$

In order to obtain an accurate determination of the azimuth constant, it is evident that all the other instrumental errors must be well determined. The value of $a$ will be determined best when the denominator $A_1 - A_2$ is made as large as possible. This condition will be fulfilled by observing one star at lower culmination and the other at upper culmination, both as near the pole as possible.

Example:- For this determination, observations were made on 43 H Cephei and Groombridge 2001. As the instrument was reversed on each star, the collimation constant was eliminated and need not be considered.

Nov. 23, 1904

| Star | T | Dec. | B.L. | Sum | Corr'd T | α | Constants |
|---|---|---|---|---|---|---|---|
| 43 H. Cephei | $00^h 56^m 7.^s14$ | -0.22 | +0.24 | +0.02 | 7.16 | $00^h 55^m 56.^s76$ | $b = +0.025$ |
| Grw. S.P. 2001 | $01^h 23^m 43.^s27$ | +0.06 | -0.03 | +0.03 | 43.30 | $01^h 23^m 36.^s97$ | |

| Star | δ | B | A |
|---|---|---|---|
| 43 H. Cephei | +85° 45' 01" | +9.44 | -9.65 |
| Grw. 2001 | +75° 53' 04" | -1.33 | +3.13 |

$\alpha_1 - T_1 = -6.33$ $\qquad$ $\alpha_2 - T_2 = -10.40$

$$a = \frac{-6.33 + 10.40}{3.13 + 9.65} = 0^s.32$$

9. Clock Correction — As the relation, which exists between the time and the instrumental errors of a transit, has already been determined, it would be well at this point to illustrate the record and reduction of a set of transit observations by means of which an ordinary determination of time is obtained. The necessary data are the date; b, the measured inclination of the horizontal axis; and T, the time that the chronometer shows for the transit of the star. The observed times of transit given are each the mean of the observed times of transit of the given star over five wires and in the reduction the collimation constant, c, is assumed to refer to the mean of these wires. This assumption can be made because each star was observed over exactly the same set of wires

In our determination of the clock correction, we get $a = +0^s.37$, $c = +0^s.54$ and $\Delta T = -7^s.10$. Now, if we use the values of the constants, $a = +0^s.34$ and $c = +0^s.50$, obtained by direct determination on the same evening, we get $\Delta T = -7^s.09$. This comparison shows that all three quantities can be determined at the same time with fairly accurate results.

Clock Correction

Nov. 23, 1904

| | Star | Clamp | T | Di. Alt. | A a | B b | C c | Sum | Corr'd T | α | Δ T | Constants |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | 41 H.C. Pher | W | 23h 43m 29.86 | −0.04 | −0.44 | +0.08 | +1.40 | +1.00 | 43m 30.86 | 23h 43m 25.72 | −7.14 | b = +0.038 |
| (2) | W Piscium | W | 23h 54m 32.16 | −0.02 | +0.21 | +0.03 | +0.55 | +0.77 | 54m 32.93 | 23h 54m 25.86 | −7.07 | a = +0.37 |
| (3) | i Ceti | E | 00h 14m 42.50 | −0.02 | +0.25 | +0.02 | −0.53 | −0.27 | 14m 42.23 | 00h 14m 35.11 | −7.12 | c = +0.54 |
| (4) | ξ Cassiop. | E | 00h 31m 49.06 | −0.03 | −0.14 | +0.06 | −0.90 | −1.01 | 31m 48.05 | 00h 31m 41.79 | −7.06 | ΔT = −7.10 |

| | δ | B | A C |
|---|---|---|---|
| (1) | +61° 17′ | +1.30 | ΔT − 1.19a + 1.59c = −6.18 |
| (2) | + 6° 20′ | +0.85 | ΔT + 0.56a + 1.01c = −6.31 |
| (3) | − 1° 21′ | +0.66 | ΔT + 0.77a − 1.01c = −7.39 |
| (4) | +57° 28′ | +1.63 | ΔT − 0.39a − 1.68c = −8.10 |

From

(1) − (2) − (3) + (4)

− 2.91a + 0.91c = − 0.58

From

− (1) − (2) + (3) + (4)

1.01a − 6.29c = − 3.00

or

a = + 0.37  c = 0.54

Clock Correction

Nov. 23, 1904

(using values of a and c obtained by direct determination)

| Star | Clamp | T | Di. Alt. | A a | B b | C c | Sum | Corr'd T | α | Δ T | Constants |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 41 H Cephei | W | 23h 43m 29.86 | −0.04 | −0.33 | +0.08 | +1.29 | +0.95 | 43m 30.81 | 23h 43m 25.72 | −7.09 | c = +0.50 |
| W Piscium | W | 23h 54m 32.16 | −0.02 | +0.18 | +0.03 | +0.51 | +0.70 | 54m 32.86 | 23h 54m 25.86 | −7.00 | b = +0.038 |
| i Ceti | E | 00h 14m 42.50 | −0.02 | +0.25 | +0.02 | −0.51 | −0.25 | 14m 42.25 | 00h 14m 35.11 | −7.14 | a = +0.32 |
| ξ Cassiop | E | 00h 31m 41.56 | −0.03 | −0.12 | +0.06 | −0.84 | −0.93 | 31m 48.93 | 00h 31m 41.79 | −7.14 | ΔT = −7.09 |

10 Conversion of Time — If the clock correction for a chronometer keeping mean solar time is desired, it can only be found by comparing the chronometer, the clock correction of which is desired, with the sidereal chronometer, the clock correction of which has already been determined. Knowing the correction for the sidereal chronometer, we can convert sidereal time into the equivalent mean solar time and then by comparison find the clock correction for the chronometer keeping mean solar time. An illustration of the actual process is given below. The reader is referred to Comstock's "Field astronomy for Engineers" for a complete explanation of the process of converting sidereal into mean solar time.

Example;— The Hornby chronometer kept sidereal time and the Bliss chronometer, standard time.

Nov. 23, 1904

| | | |
|---|---|---|
| Bliss Chronometer | = | $9^h\ 45^m\ 20^s.0$ |
| Hornby Chronometer | = | $2^h\ 05^m\ 10^s.5$ |
| $H \Delta T$ | = | $-7^s.10$ |
| $\theta$ | = | $2^h\ 05^m\ 03^s.40$ |
| $\alpha_1$ | = | $16^h\ 09^m\ 04^s.43$ |
| $\theta - \alpha_1$ | = | $04^h\ 55^m\ 08^s.97$ |
| $(\theta - \alpha_1)\ a''$ | = | $01^m\ 37^s.90$ |
| M | | $09^h\ 53^m\ 31^s.07$ |
| E. of 90th Mer. | = | $07^m\ 06^s.09$ |
| M (90th Mer.) | = | $09^h\ 46^m\ 24^s.98$ |
| $B \Delta T$ | = | $+01^m\ 04^s.98$ |

11. Precision of Results :- If there were no [illegible] made by the observer, we should always get the same value for the wire intervals when reduced to equatorial time interval. When this is done, many different values are obtained for the same wire interval. From these different values the probable error of the value of a wire interval may be found, as well as the probable error of a single transit. This will give some knowledge of the precision of the results.

Example:- The data was obtained from observations made on thirteen stars but only enough data will be given to show the arrangement and the method by which the probable error of a single transit is obtained.

| Star | Wire | T | I | $v - \delta$ | $I \cos \delta$ |
|---|---|---|---|---|---|
| $\sigma$ Andromeda | $C_1$ | 26.3 | 8.9 | 0.745 | 6.63 |
| | $C_2$ | 22.4 | 5.0 | | 3.72 |
| | $C_3$ | 17.4 | | | |
| | $C_4$ | 13.0 | 4.4 | | 3.28 |
| | $C_5$ | 8.7 | 8.7 | | 6.48 |
| $\zeta^2$ Aquarii | $C_1$ | 16.9 | 8.9 | 0.929 | 8.27 |
| | $C_2$ | 11.0 | 4.0 | | 3.72 |
| | $C_3$ | 07.0 | | | |
| | $C_4$ | 04.3 | 2.7 | | 2.51 |
| | $C_5$ | 01.0 | 6.0 | | 5.57 |
| $\kappa$ Pisc. Austr. | $C_1$ | 17.0 | 7.1 | 0.865 | 7.01 |
| | $C_2$ | 13.0 | 4.1 | | 3.55 |
| | $C_3$ | 08.9 | | | |
| | $C_4$ | 05.1 | 3.8 | | 3.29 |
| | $C_5$ | 01.1 | 7.8 | | 6.75 |
| $\iota$ Andromeda | $C_1$ | 00.4 | 9.9 | 0.784 | 7.27 |
| | $C_2$ | 05.1 | 5.2 | | 3.82 |
| | $C_3$ | 10.3 | | | |
| | $C_4$ | 15.1 | 4.8 | | 3.52 |
| | $C_5$ | 11.3 | 1.0 | | 6.61 |

Now, if we take the mean of the values found for each wire interval and form residuals, we can compute the probable error in the value of any one wire interval. Since the wire interval is merely the difference of time between the transits of the star over the given wire and the middle wire, we are able to find the probable error of any one transit

| $C_1$ | | $C_2$ | | $C_4$ | | $C_5$ | |
|---|---|---|---|---|---|---|---|
| I cos δ | v | I cos δ | v | I cos δ | v | I cos δ | v |
| 6.63 | 0.44 | 3.72 | 0.09 | 3.28 | 0.00 | 6.48 | 0.05 |
| 8.27 | 1.20 | 3.72 | 0.09 | 2.51 | 0.77 | 5.57 | 0.86 |
| 7.01 | 0.06 | 3.55 | 0.08 | 3.29 | 0.01 | 6.75 | 0.32 |
| 7.27 | 0.20 | 3.82 | 0.19 | 3.52 | 0.24 | 6.61 | 0.18 |
| 7.05 | 0.02 | 3.67 | 0.04 | 3.86 | 0.58 | 6.86 | 0.43 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| mean = 7.07 | [+v] = 4.11 | mean = 3.63 | [+v] = 1.99 | mean = 3.28 | [+v] = 2.97 | mean = 6.43 | [+v] = 3.55 |

As we have fifty-two residuals and four quantities involved

$$r_I = \frac{0.845\,(12.62)}{\sqrt{52 \times 48}}$$ (probable error of a single wire interval)

The probable error of a single transit would be

$$r = \frac{0.845\,(12.62)}{\sqrt{2}\,\sqrt{52 \times 48}} = 0^s.15$$

The real test for accuracy and care in observing

is the agreement of the various values of $\Delta T$ determined from observations made on different stars on the same night. There was not enough data to compute the probable error from those results.

Neither this apparent precision nor the precision that would be obtained from comparing the different values of $\Delta T$, is completely reliable, for nearly all observers possess individual peculiarities, called personal equation, which cause them to observe all stars either too soon or too late by an almost constant amount. No indication of the presence or magnitude of this constant personal error can be detected by determining the probable error of a single transit. The personal equation, although often a considerable source of error, is of no practical consequence unless the observations of different persons are to be combined. In such cases, the personal equation of each observer must be determined.

Various devices are employed to determine the amount of the personal equation of an observer. In this determination, a so-called personal-equation machine was used. A fine point of light moved back and forth across a set of wires and its transits were recorded automatically by a chronograph. The observer also records the transits by means of a key connected with the chronograph. The difference between the two records was measured and in that way the personal equation was determined.

Twenty sets of observations were made but only

a few will be given to show the arrangement of the data and the method of the determination. The probable error of the personal was determined in order to see how reliable the value of the personal equation really was.

Wed. Dec. 14, 1904

| R to L | v | L to R | v | R to L | v | L to R | v |
|---|---|---|---|---|---|---|---|
| 1.09 | 9 | 1.15 | 8 | 1.11 | 1 | 0.95 | 6 |
| 1.10 | 1 | 1.10 | 3 | 1.09 | 3 | 1.20 | 19 |
| 1.22 | 4 | 1.11 | 4 | 1.15 | 3 | 1.06 | 5 |
| 1.16 | 2 | 0.98 | 9 | 1.12 | 0 | 0.84 | 17 |
| 1.33 | 15 | 1.07 | 1 | 1.12 | 0 | 1.02 | 1 |
| 1.18 | 38 | 1.068 | 31 | 1.118 | 7 | 1.014 | 48 |
| | 0.112 | | | | 0.104 | | |
| P.E. | 0.056 | | | P.E. | 0.052 | | |

In this way twenty values were obtained for the personal equation and by taking their mean

Personal Equation = 0.s06 Eata

and found that the

Probable error of the personal equation due to any one set

$$= \frac{0.845 \times 0.1}{\sqrt{20 \times 19}} = 0.013 \quad (0.0127)$$

Probable error of the result $= \frac{0.013}{\sqrt{20}} = 0.003$

Then by forming residuals for each single observation in a set, we find that the

Probable error of any single observation $= \frac{0.845 \times 8.78}{\sqrt{200 \times 160}} = 0.04$

Probable of any single transit $= \frac{0.04}{\sqrt{2}} = 0.029$

Probable error of the mean of any set = $\frac{0.04}{\sqrt{5}} = 0.019$

Probable error of the difference between any pair of means

$= \sqrt{2} \times 0.019 = 0.0266$

Probable error of Personal Equation due to any ~~set~~ set

$= \frac{0.0266}{2} = 0.013 \qquad (0.0133)$

Probable error of result = $\frac{0.013}{\sqrt{10}} = 0.003$

The probable error of the result was found to be the same by two methods of computation, therefore

Personal Equation = $0^s.06 \pm 0^s.003$ (Cate.)

12. Zenith Telescope - Description — As the transit instrument has a sensitive spirit level attached at right angles to the rotation axis, and the micrometer box can be rotated so that the micrometer wire moves parallel to this axis, it is also a zenith telescope. The level is called a zenith level.

13. Determination of Geographical Latitude — The latitude may be determined quite accurately by means of the zenith telescope. Talcott's method is the one employed and consists in measuring the difference of the zenith distances of two stars, one of which culminates south of the zenith and the other north of the zenith. This difference should not be much larger than one half the diameter of the field of view, to avoid observing too near the edge of the field. The difference of the right ascensions of the two stars should not exceed twenty minutes, to avoid any change in the constants of the instrument between the two halves of the observation, and should never be so small as to cause undue haste. The zenith distances should always be less than 35° to avoid uncertainty in refraction.

If we let $\delta'$, $z'$ and $\delta''$, $z''$ be the declinations and zenith distances of the south and north star respectively, then

$$\delta' = \Phi - z'$$

$$\delta'' = \Phi + z''$$

Therefore

$$\delta' + \delta'' = 2\Phi + (z'' - z')$$

where $z'' - z' < 40'$, is the condition which the pair of stars must fulfill for this instrument.

In this particular case, the approximate latitude was 40°06' and the instrument's field of view was 60' in diameter. The form of the observing list was, as follows;

| Star | Mag | AR | δ | z | setting | micrometer |
|---|---|---|---|---|---|---|
| Δ Ursae Maj | 3.3 | $10^h11^m19^s$ | +43°24' | 03°18' | N 29°16' | 13' up |
| 31 Leon min. | 4.3 | $10^h22^m20^s$ | +37°12' | 02°54' | S 29°16' | 13' down |

To observe a star, the "setting" is made as described for the use of the transit, then the zenith level is unclamped and rotated until the bubble plays and then it is re-clamped. The bubble is brought near the middle of the tube by means of a screw at one extremity of the tube. Then, the micrometer wire is moved to the part of the eye-piece, where it is known the first star will pass. At the time of culmination or within a few seconds of this time, bisect the star with the micrometer wire and read the zenith level and the micrometer. Reverse the instrument gently, bring the bubble back to the center again by means of the tangent movement and observe the second star in the same manner as the first.

As the telescope must be shifted in a majority of cases when the instrument is reversed, extreme care was required to prevent a change in the angle between the position of the level and the line of sight during the progress of an observation. This precaution must be presumed.

Let $m_0$, be the micrometer reading of some point in the field assumed as the micrometer zero: $z_0$ the apparent zenith distance for the reading $m_0$, when the bubble is at the center of the zenith level tube; $m'$, $m''$ the micrometer readings on the two stars, the readings being supposed to increase with the zenith distance; R, the value of a revolution of the micrometer screw; $b'$, $b''$, the level constants for the two stars, plus when the north end is high. The true zenith distance of the southern star is then

$$z' = z_0 + (m' - m_0) R + b' + c' ;$$

and of the northern star

$$z'' = z_0 + (m'' - m_0) R - b'' + c''$$

Since

$$\delta' + \delta'' = 2\Phi + (z'' - z')$$

we obtain by solving for $\Phi$

$$\Phi = \tfrac{1}{2}(z' + \delta'') + \tfrac{1}{2}(m' - m'') R + \tfrac{1}{2}(b' + b'') + \tfrac{1}{2}(c' - c'')$$

As the zero of the level scale is at one end of the tube

$$\tfrac{1}{2}(b' + b'') - \tfrac{1}{2}[\pm(n' - s'') \pm (s' - n')]\, d$$

the upper sign being used when $n'$ is greater than $s'$, the lower when the reverse is true.

The refraction correction is small and may be taken from tables which have been computed for this purpose. The following table - due to Campbell, was employed by the writer. The sign of this correction is the same as that of the micrometer correction

Values of $\frac{1}{2}(z' - z'')$

| $\frac{z'-z''}{2}$ | z=0° | z=10° | z=20° | z=25° | z=30° | z=35° |
|---|---|---|---|---|---|---|
| 0' | ".00 | ".00 | ".00 | ".00 | ".00 | ".00 |
| 1' | .02 | .02 | .02 | .02 | .02 | .02 |
| 2' | .03 | .03 | .04 | .04 | .04 | .05 |
| 3' | .05 | .05 | .06 | .06 | .07 | .08 |
| 4' | .07 | .07 | .08 | .08 | .09 | .10 |
| 5' | .08 | .09 | .10 | .10 | .11 | .13 |
| 6' | .10 | .11 | .11 | .12 | .13 | .15 |
| 7' | .12 | .12 | .13 | .14 | .15 | .18 |
| 8' | .13 | .14 | .15 | .16 | .17 | .21 |
| 9' | .15 | .16 | .17 | .18 | .20 | .23 |
| 10' | .17 | .18 | .19 | .21 | .23 | .26 |
| 11' | .18 | .20 | .21 | .23 | .25 | .29 |
| 12' | .20 | .21 | .23 | .25 | .27 | .31 |

The arrangement and reduction of the data are given on the next page. K = 56".28 and λ = 0".357

| Star | Level N | Level S | R | | δ |
|---|---|---|---|---|---|
| o Ursae maj | 21.3 | 63.8 | 62.750 | | 61° 02′ 13″.1 |
| δ Canori | 72.1 | 30.0 | 19.784 | | 18° 30′ 03.1 |
| | +8.50 | | 42.966 | δ′+δ″ | 79° 32′ 16″.2 |
| | | | | ½(b′+b″) | + 06.1 |
| | | | | mic | +40 12.1 |
| | | | | Refn | + 00.8 |
| | | | | 2Q | 80° 12′ 41.2 |
| | | | | Q | 40° 06′ 20″.6 |
| 36 Lyncis | 39.1 | 66.8 | 24.543 | | 43° 36′ 34″.1 |
| 38 Lyncis | 10.0 | 62.5 | 63.296 | | 37 12 14.8 |
| | +23.0 | | 38.753 | δ′+δ″ | 80 48 48.9 |
| | | | | ½(b′+b″) | + 16.4 |
| | | | | mic | −36 21.0 |
| | | | | Refn | − 0.6 |
| | | | | 2Q | 80 12 43.5 |
| | | | | Q | 40° 06′ 21″.65 |
| λ Ursae maj. | 69.8 | 40.0 | 55.610 | | 43° 23′ 18″.0 |
| 31 Leon min. | 33.9 | 63.8 | 31.856 | | 37° 11′ 36″.5 |
| | +6.05 | | 23.754 | δ′+δ″ | 80° 34′ 54″.5 |
| | | | | ½(b′+b″) | + 4.3 |
| | | | | mic. | −22 16.9 |
| | | | | Refn | − 0.4 |
| | | | | 2Q | 80° 12′ 41″.5 |
| | | | | Q | 40° 06′ 20″.75 |
| 35 H Ursae maj | 21.8 | 66.8 | 28.587 | | 69° 34′ 27″.6 |
| 1 Leonis | 64.8 | 62.5 | 54.811 | | 11° 02′ 42″.1 |
| | +12.8 | | 26.284 | δ′+δ″ | 80° 37′ 10″.3 |
| | | | | ½(b′+b″) | + 9.1 |
| | | | | mic | −24 39.2 |
| | | | | Refn | − 0.1 |
| | | | | 2Q | 80° 12′ 39.5 |
| | | | | Q | 40° 06′ 19.7 |

Mean Q = 40° 06′ 20″.1

Previous value mean Q = 40° 06′ 20″.1

14. Reduction from mean to apparent Place. — It is often found necessary to reduce the mean place of a star to apparent place before it may be employed for certain determinations. The mean places of stars are given in the Berliner astronomisches Jahrbuch. Their apparent places may be calculated by the method given in the American Ephem... pp. 566-569.

Apparent places, April 11, 1905.

| | 36 Lyncis | 38 Lyncis | 35 H Ursae Maj. |
|---|---|---|---|
| $\alpha_0$ | 136° 54′ | 138° 01′ | 159° 04′ |
| G | 72° 46.4 | 12° 36.4 | 72° 43.7 |
| $G+\alpha_0$ | 209° 40.4 | 210° 31.4 | 231° 47.7 |
| $H+\alpha_0$ | 383° 44.4 | 384° 51.4 | 405° 47.7 |
| $\delta_0$ | 43° 37.0 | 37° 12.0 | 69° 34.0 |
| log g | 0.13444 | 0.93494 | 0.93500 |
| log cos$(G+\alpha_0)$ | 9.93895$_n$ | 9.93411$_n$ | 1.19132$_n$ |
| log $(g')$ | 0.87389$_n$ | 0.86971$_n$ | 0.72632$_n$ |
| log h | 1.27855 | 1.27855 | 1.27859 |
| log cos$(H+\alpha_0)$ | 9.96160 | 9.95718 | 9.84338 |
| log sin $\delta_0$ | 1.83364 | 1.78154 | 1.94179 |
| log $(h')$ | 1.07884 | 1.01785 | 1.09376 |
| log i | 0.8793$_n$ | 0.8793$_n$ | 0.8190$_n$ |
| log cos $\delta_0$ | 1.86197 | 9.90118 | 9.54284 |
| log $(i)$ | 0.73901$_n$ | 0.18048$_n$ | 0.42184$_n$ |
| $\delta'_0$ | 43° 36′ 35.″02 | 37° 12′ 17.80 | 69° 34′ 23.″12 |
| $(g')$ | − 7.48 | − 1.41 | − 5.″33 |
| $(h')$ | + 11.99 | +10.42 | + 12.41 |
| $(\iota)$ | − 5.48 | − 6.03 | − 2.64 |
| $\delta$ | 43° 36′ 34.″05 | 37° 12′ 14.″78 | 69° 34′ 27.″56 |
| H = | 246° 50′.4 | 246° 50′.4 | 246° 43′.7 |

## References

Berliner astronomisches Jahrbuch, 1905 Pp. 149-312.

American Ephemeris, 1905 Pp. 290-400, 566-569.

Campbell - "Elements of Practical astronomy"
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Chauvenet - "Astronomy" Pp. 131-282, 340-367

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Pp. 40-44, 99-111, 155-195.

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